Low Endurance Fatigue in Metals and Polymers

Part 1 *Stress/Strain Relationships*

B. TOMKINS*, W. D. BIGGS
Engineering Laboratory, University of Cambridge, UK

The low endurance fatigue behaviour of commercial purity aluminium, Nylon 66 and an epoxy resin is examined and compared. Both aluminium and nylon obey a plastic strain criterion for failure and this behaviour is independent of frequency. No satisfactory criterion for failure could be found for the epoxy resin whose behaviour is strongly frequency-dependent.

1. Introduction

In recent years an increasing amount of work has been devoted to a study of the fatigue behaviour of metals in the low endurance range (1 to 10^5 cycles). This has led to a better understanding of basic fatigue processes and has yielded important data for use in the design of various structures and components which are cyclically stressed into the plastic range. The approach has, generally, been phenomenological and has tried to relate the fundamental variables (stress, strain, cycles, temperature, frequency, etc.) and to assess their effect on the initiation and propagation of a crack.

Most fatigue studies on polymers have been of the empirical type in the high endurance range ($> 10^5$ cycles). Some polymers appear to show a fatigue limit in stress-cycling tests but there is some doubt whether the fatigue process involves general structural disintegration or a process of crack initiation and propagation, as in metals. Very little work has been done in the low endurance range. The most detailed examination is probably that due to Gent *et al* [1]. These authors found that, in many rubbers, a modified Griffith criterion governs crack growth throughout the fatigue range.

The present work examines the low endurance fatigue behaviour of aluminium, Nylon 66 and Araldite epoxy resin. The fatigue process is first considered phenomenologically; later papers will discuss cumulative damage and crack initiation and propagation.

2. Materials

Tensile true stress/true strain curves of the materials used are shown in fig. 1, the relevant mechanical properties are summarised in table I.
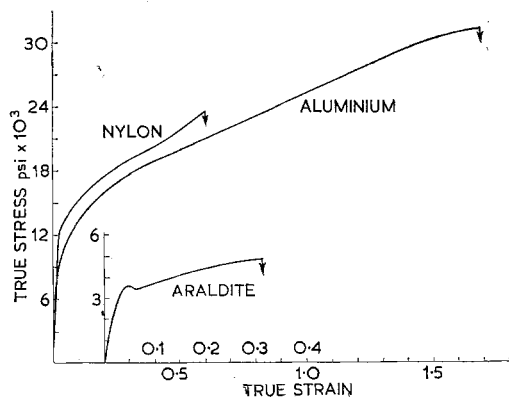

Figure 1 Tensile true stress/true strain curves of materials used. (Note change of scale for Araldite.)

The aluminium was of commercial purity and, in the annealed condition, had an average grain size of 0.20 mm. Nylon 66 is a semicrystalline polymer; the crystallinity of the annealed commercial quality material used in the present tests was about 70%. There is still some disagreement about the structure of semicrystalline

*Now at UKAEA (Reactor Group), Springfield, Lancs, UK.

TABLE I Typical mechanical properties of materials used.

	CP aluminium	Nylon 66	Araldite epoxy resin
Initial condition	Annealed	Annealed	As-cast
Young's modulus (psi × 10⁶)	10.2	0.46	0.153
Tensile yield stress (0.2% offset) (psi × 10³)	10.0	12.0	—
True fracture stress (psi × 10³)	31.3	24.0	4.9
True fracture strain	1.69	0.58	0.305
Poisson's ratio	0.32	0.42	0.47
Vickers Hardness No.	26	—	—

polymers but Zaukelies [2] has shown that highly crystalline Nylon 66 can deform by dislocation-controlled slip on a closely defined slip system. Recent work by Predecki and Statton [3] has examined the possible dislocations formed by chain ends in this material.

Araldite epoxy resin has a highly cross-linked network structure and is usually very brittle. A more flexible structure can be obtained by the addition of a plasticiser such as polysulphide rubber which, in the form of long chain molecules, bonds to the epoxide groups.

3. Test Specimens and Experimental Procedure

The present tests followed other investigations in cycling between equal tensile and compressive strain limits. A modified 6 ton Losenhausen fatigue machine allowed manual operation at 5 c/m with automatic operation at 300 c/m. Load readings at any part of a cycle were obtained by means of strain gauges embedded in the specimen grips. The specimen was cylindrical, waisted at the central section in a radius of 3.8 cm to a minimum diameter of 0.76 cm. Strain control in this minimum section was achieved by means of variable inductance or capacitance transducers measuring diameter changes; these were converted into true plastic and elastic strains by means of the relevant material constants. By this means hysteresis loops relating load and diameter were displayed on a C R O and, from these traces, true stress/true strain loops were derived.

4. Fatigue Hardening and Softening

Many investigators have found that, when an annealed metal is fatigued it hardens, whereas if

it be initially hard (either metallurgically or because of deformation), it softens. In strain-cycling tests the resistance of a material to cyclic deformation is shown by the variation of stress range with cycles.

4.1. Aluminium

Stress range versus number of cycles is plotted in fig. 2. The initial rate of hardening is rapid and then decreases; this lasts for some 10 to 20% of the life, thereafter a steady rate of hardening is attained and this continues up to the point of maximum stress range just before failure. The rapid drop in stress range preceding failure is attributed to the formation of cracks reducing the section on which the true stress is computed.
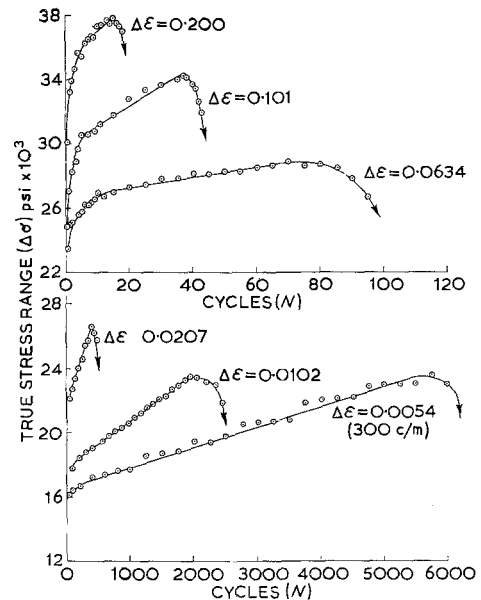

Figure 2 True stress range versus cycles for annealed aluminium. Cycling frequency 5 c/m except where shown.

Over the steady range we can define a strain-hardening rate for a particular strain range as

$$\gamma = \mathrm{d}(\Delta\sigma)/\mathrm{d}N \qquad (1)$$

In fig. 3 γ has been plotted against $\Delta\epsilon_p$; this gives a linear relationship of the form

$$\gamma = K\Delta\epsilon_p^n \qquad (2)$$

Note that the curve includes points obtained from tests at both 5 and 300 c/m indicating that the hardening effect may be independent of frequency. The strain-hardening exponent (deduced from the tensile stress/strain relationship $\sigma = K_1\epsilon^n$) is plotted at a value of $\epsilon_p = 2\epsilon$ and lies well above the fatigue line.
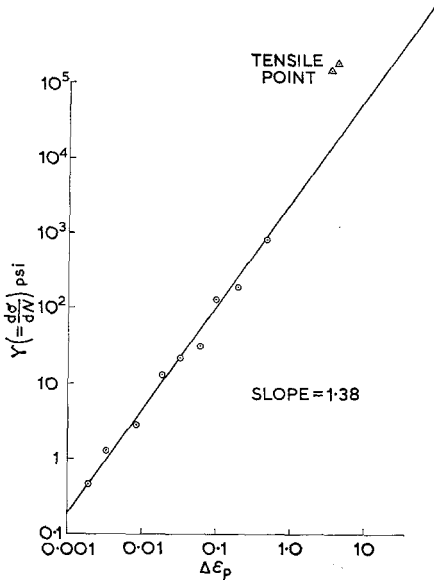
533

5

Figure 3 Cyclic strain hardening rate versus plastic strain for aluminium.

Figure 5 Aluminium – true stress range at half-life versus plastic strain range. Circles taken directly from the cyclic stress/strain curves at half-life; triangles are derived points and represent the saturation hardness value at low cyclic strains.

Fig. 4 shows that, at higher strains, hardening occurs at a decreasing rate during the test whereas, at lower strains, there is a "plateau" of non-hardening behaviour before hardening starts again. It also suggests that a limiting stress range is reached in all specimens with lives > 2000 cycles.
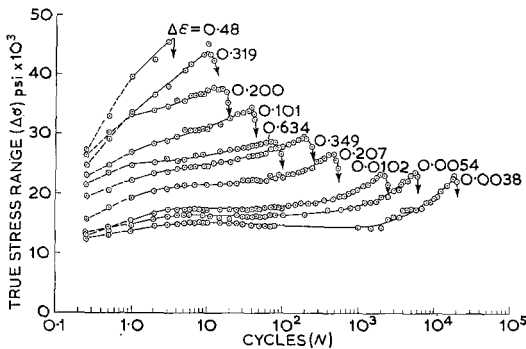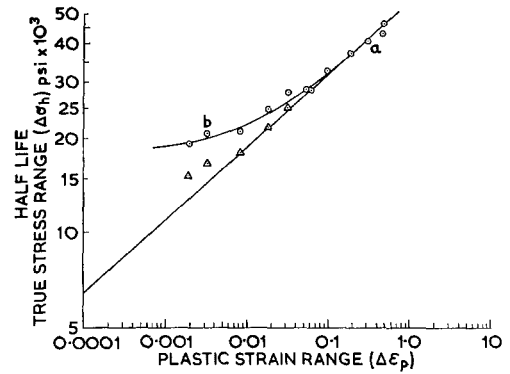
and if this line be extended the "plateau" stress ranges of fig. 4 lie on the extrapolation. At low strains the line levels off to a limiting value (b).

4.2. Nylon

Two batches of nylon were tested, these showed slight differences in properties and are designated Batch I and Batch II – most tests were carried out on Batch II material.
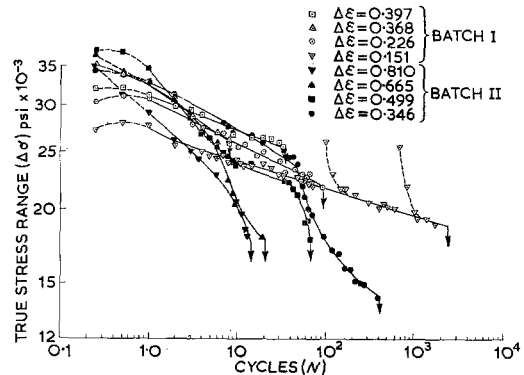

Figure 4 Aluminium – true stress range versus cycles for different plastic strain ranges.

Figure 6 Nylon 66 – true stress range versus cycles. Dashed curves show recovery as a result of holding the specimen at zero load during the test.

Following the procedure adopted by Smith et al [4], Biggs and Topper [5] and others, the cyclic stress range/strain range curve ($\Delta\sigma$ versus $\Delta\epsilon_p$) is plotted in fig. 5 for the value of $\Delta\sigma$ at half the specimen life. The curve is linear at high strains (a), and may be expressed in the form

$$\Delta\sigma_h = a\Delta\epsilon_p{}^\beta \qquad (3)$$

Fig. 6 shows the variation of stress range with number of cycles; the most notable feature is that after the first cycle $\Delta\sigma$ decreases continuously until fracture supervenes. Batch II specimens were more sensitive to rate of straining as shown by the rapid acceleration in stress

relaxation at about 50 cycles when the cycling rate was increased. The dashed curves in fig. 6 show the effect of holding the specimen under zero load during the test. As a result of recovery the specimen shows a value of $\Delta\sigma$ almost as high as that obtained in the first cycle. This suggests that no gross deterioration occurs and appears to support the contention of Prevorsek and Lyons [6] that fatigue in semicrystalline polymers is basically a process of crack growth and not of general structural disintegration. However the fact that, after stoppage, the original $\Delta\sigma$–N curve is eventually rejoined indicates some permanent effect of the previous cycles.

Fig. 7 shows the effect of cycling frequency – the first 50 cycles were at 1.5 c/m, the next 50 at 5 c/m and thereafter at 300 c/m. For the longer
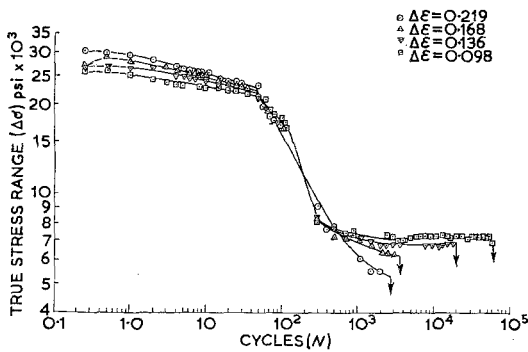
life specimens $\Delta\sigma$ soon levels off to a constant value which is maintained until fracture; for shorter life specimens the stress range decreases continuously until fracture, as in the tests at lower frequency. The order of the curves is reversed as the cycling changes from slow to fast so that, during fast cycling, the highest stress range is being maintained by the specimen at the lowest strain range.

In considering the physical processes reflected in the curves of figs. 6 and 7 it is useful to consider the form of the hysteresis loop at various points. Fig. 8a shows a typical loop with both linear and non-linear elastic behaviour as well as true plastic (irreversible) deformation. The width of the loop at zero stress ($\Delta\epsilon_p$ for metals) varies considerably with strain rate, making it impossible to determine the true value of $\Delta\epsilon_p$. Fig. 8b shows the effect of frequency; the reduction in $\Delta\sigma$ is due primarily to a decrease in modulus caused by hysteretic heating; this explains the reversal of order of $\Delta\sigma$ in the fast cycled specimens (fig. 7). At higher strain ranges the heating is greater and hence the modulus drop is greater, so that the less strained specimens have higher stress ranges under fast cycling. Fig. 8c shows that the decrease in $\Delta\sigma$ with cycles is due, primarily, to a decrease in flow stress rather than in modulus. The recovery effects noted in fig. 6 are thus probably due to an increase in flow stress, i.e. in the resistance to plastic deformation.
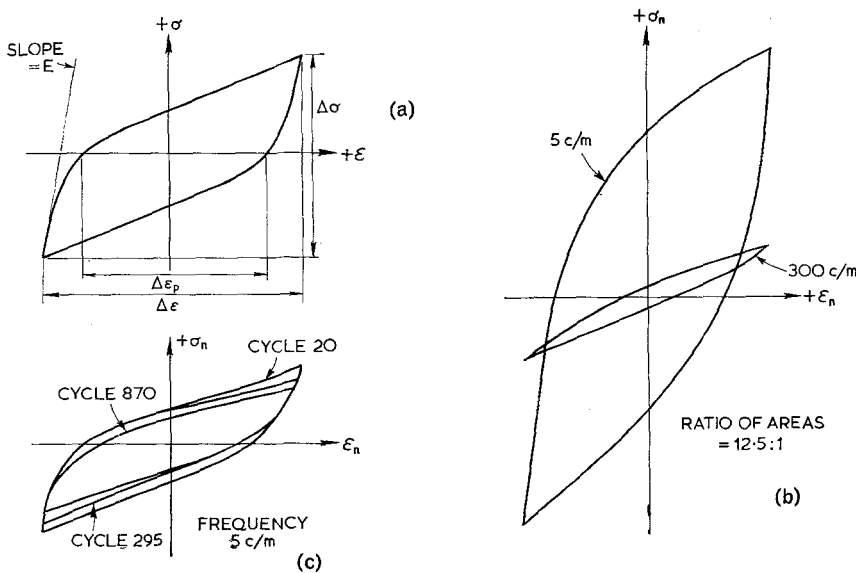

Figure 7 Nylon 66 – true stress range versus cycles. Batch II material, cycled at 5 c/m for first 100 cycles, thereafter at 300 c/m.

Figure 8 (a) Typical hysteresis loop for nylon, (b) effect of frequency, (c) decrease in stress range during test.
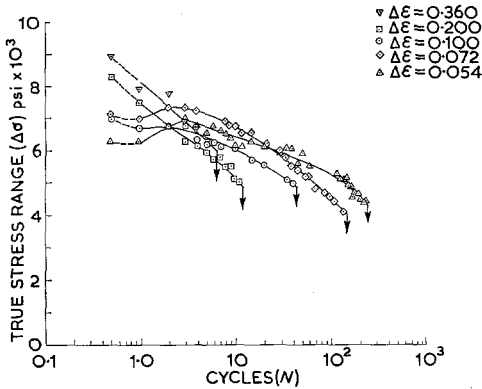
535

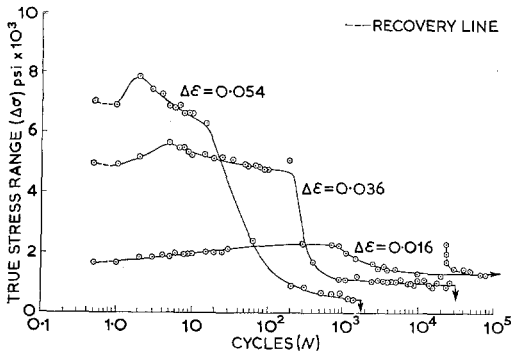Figure 9 Araldite – true stress range versus cycles at 5 c/m

Figure 10 Araldite – true stress range versus cycles at 300 c/m (first 100 cycles at 5 c/m).

4.3. Araldite

Figs. 9 and 10 show that the behaviour of Araldite is similar to that of nylon. A similar cycling procedure was followed and the reversal of stress range level is again observed, as also is recovery. The indications are that gross structural deterioration does not occur with cycling.

5. Strain Behaviour

Manson [7], Coffin [8, 9] and others have suggested that the low endurance behaviour of metals is governed by an expression of the form

$$\Delta\epsilon_p N_f^{\alpha} = C \qquad (4)$$

where the exponent α may lie between 0.5 and 0.7 and the constant C is related to the true fracture strain in tension.

For aluminium it was possible to determine $\Delta\epsilon_p$ from the measured diametral strain range $\Delta\epsilon_d$ and the true stress range using the appropri-

536

ate material constants. This was not possible for the polymers since the true plastic and visco-elastic strains could not be separately measured. Thus all strain relationships for polymers are defined in terms of the *total* strain range $\Delta\epsilon$.
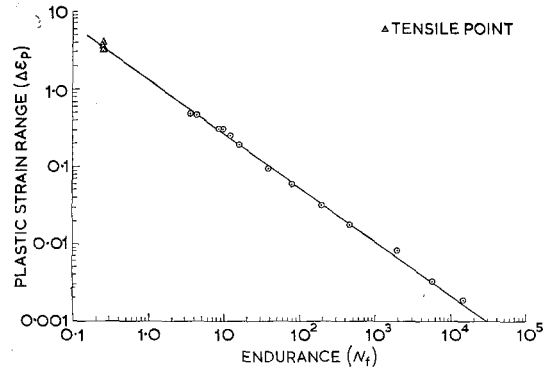

Figure 11 Aluminium – plastic strain range versus endurance.

5.1 Aluminium

Fig. 11 shows plastic strain range versus endurance, "endurance" being taken as the number of cycles before the stress decreased rapidly due to cracking. This is usually only slightly less than the number of cycles for total separation. The appropriate values here for equation 4 are $\alpha = 0.693$ and C= 1.32. The value for α compares well with $\alpha = 0.688$ obtained by Smith et al [4]. The tensile point is plotted in fig. 11 as twice the fracture strain at an endurance of 1/4 cycle [5]. The data show no apparent dependence on frequency.
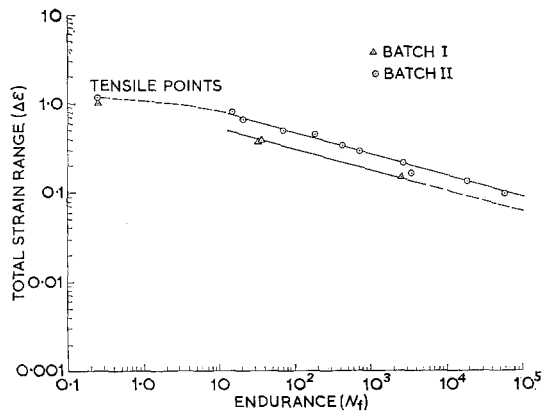

Figure 12 Nylon 66 – total strain range versus endurance.

5.2. Nylon

Results for both Batch I and Batch II materials are shown in fig. 12. There is little scatter and the curves are nearly log-linear over most of the range. The tensile point does not lie on the curve but it may lie on the non-linear portion of the curve associated with the results from specimens where $N_f < 100$ cycles.

A Coffin-Manson type relationship exists for the linear part of the curve; for this material $\alpha = 0.23$ and C $= 0.64$. As noted above, this is derived on a basis of total strain but it was observed that the breadth of the hysteresis loop (apparent $\Delta\epsilon_p$) was directly proportional to the total strain range at both frequencies. Thus it might be expected that if the true plastic strain range could be measured then it too would be proportional to the total strain range and hence that a Coffin-Manson relationship with the same exponent would exist between $\Delta\epsilon_p$ and N_f.
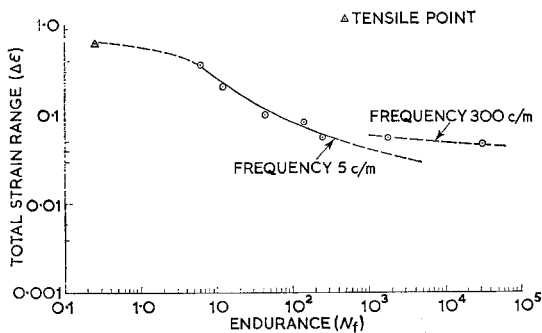

Figure 13 Araldite – total strain range versus endurance.

5.3. Araldite

Fig. 13 shows that the results for Araldite lie on two distinct curves according to the cycling frequency. In tests performed at the same strain range the test at 300 c/m showed an endurance 7.5 times that obtained at 5 c/m. Further the curves are non-linear so that a simple Coffin-Manson relationship does not exist.

6. Discussion
6.1. Hardening and Softening

The $\Delta\sigma/N$ curves show that the bulk behaviour of the metal is quite different from that of the two polymers. Furthermore, aluminium, especially at low strains, behaves differently from other metals under similar conditions of imposed strain in that no saturation value of stress was

achieved, but rather hardening continued throughout the test. Coffin and Tavernelli [9] observed such hardening in their tests at very high strains ($N_f < 200$ cycles) on polycrystalline aluminium and Raymond and Coffin [10] have attempted to explain this continued hardening in terms of geometrical changes in a waisted specimen. They claim that the hardening is an apparent rather than a real effect. However, in the present tests, continued hardening was obtained at strain ranges which were more than an order of magnitude lower than the lowest range used by Coffin and Tavernelli and no significant change in specimen shape was observed.

An explanation of the steady hardening in aluminium has been given by Grosskreutz [11] who has related the hardening to an intensification of the subgrain structure formed during the initial, rapid hardening stage.

The similarity in form of the curves for nylon and Araldite is probably a consequence of the viscoelastic behaviour of these materials. The continued "softening" of polymeric materials under cyclic straining has been noted in hysteresis studies of rubber [12] and is thought to be due to non-affine deformation of the polymer network.

6.2. Strain Behaviour

The behaviour of both aluminium and nylon accords with the Coffin-Manson relationship. Since this relationship is, essentially, an expression of the rate of crack propagation, the similarity in results suggests a similarity in the crack propagation process in both materials (for endurances < 100 cycles in nylon). However, the exponent α for nylon is only about one-thrd that for aluminium so that a large variation in endurance is obtained for only a small variation in the applied strain range. No obvious explanation for this could be obtained in terms of crack initiation and propagation. A later paper will show that, in nylon, the main crack is established much more slowly. Recently Tomkins [13] has shown that, for metals, the Coffin-Manson exponent α, can be related to the exponent β of the cyclic stress/strain curve (equation 3) by the expression

$$\alpha = 1/(2\beta + 1) \qquad (5)$$

and, while the sensitivity to strain made it impossible to determine β and hence to check equation 5 for nylon; readings taken from the more stable stress range/cycles curves indicate that β for nylon will be considerably higher than

537

for aluminium. This would be in accordance with the observed trend in the exponent α.

The fact that the strain range/endurance relationship is independent of frequency for both aluminium and nylon suggests a similarity in crack growth processes in these materials and a different process in Araldite. The connection between the crack growth mechanism and the Coffin-Manson relationship will be examined in a later paper.

Clearly the strain energy criterion of failure (Feltner and Morrow [14]) is of doubtful use in considering the behaviour of polymers, since the area of the hysteresis loop is not equal to the plastic strain energy because of viscous work which is dissipated as heat. We consider therefore that the Coffin-Manson relationship is more useful as a general criterion of failure.

7. Conclusions

(i) Annealed aluminium hardens continuously and linearly for most of the life; this is probably due to the development of a dislocation substructure. Prestrained aluminium softened after an initial period of rapid (approximately exponential) softening.

(ii) Both annealed nylon and as-cast Araldite show continuous non-linear softening; this is probably due to the viscoelastic response to an applied strain.

(iii) Both aluminium and nylon obey a plastic strain criterion of fatigue failure, as expressed by the Coffin-Manson relationship, $\Delta\epsilon_p N_f{}^\alpha = C$. For aluminium $\alpha = 0.693$, $C = 1.32$; for nylon $\alpha = 0.237$, $C = 0.64$.

(iv) No definite strain criterion for failure of Araldite could be found.

(v) The endurance of both aluminium and nylon was independent of frequency for a given strain range. For Araldite an increase in frequency caused a marked increase in endurance.

(vi) In the polymers the strain energy/cycle is not constant for a given strain range and a criterion of failure based upon strain energy is therefore of doubtful value.

Acknowledgement

Appendix

N = Number of strain cycles at a given time
N_f = Value of N at failure.
σ = True tensile stress.
$\Delta\sigma$ = True stress range for a strain cycled specimen.
$\Delta\sigma_h$ = Value of $\Delta\sigma$ at half the life of the specimen.
ϵ = True tensile strain.
$\Delta\epsilon$ = Total true strain range.
$\Delta\epsilon_p$ = True plastic strain range (= the breadth of the hysteresis loop at $\sigma = 0$).
$\Delta\epsilon_d$ = True diametral strain range.
E = Young's modulus.
γ = Linear strain hardening rate when tested at a particular value of $\Delta\epsilon_p$.
C, K, K_1, α, β are constants.

References

1. A. N. GENT, P. B. LINDLEY, and A. G. THOMAS, *J. Appl. Polymer Sci.* **8** (1964) 455.
2. D. A. ZAUKELIES, *J. Appl. Phys.* **33** (1962) 2797.
3. P. PREDECKI and W. O. STATTON, *ibid* **37** (1966) 4053.
4. R. W. SMITH, M. H. HIRSCHBERG, and S. S. MANSON NASA Tech. Note D1574 (1963).
5. W. D. BIGGS and T. H. TOPPER, *App. Mat. Res.* **5** (1966) 131.
6. D. PREVORSEK and W. J. LYONS, *J. Appl. Phys.* **35** (1964) 3152.
7. S. S. MANSON, NASA Report 1170 (1954).
8. L. F. COFFIN JR, *Trans. ASME* **76** (1954) 931.
9. L. F. COFFIN JR and J. F. TAVERNELLI, *Trans AIMME* **215** (1959) 794.
10. M. H. RAYMOND and L. F. COFFIN JR, *Acta Metallurgica* **11** (1963) 801.
11. J. C. GROSSKREUTZ, "Fatigue—an interdisciplinary approach", edited by Burke (Syracuse University Press, 1964) p. 27.
12. K. A. GROSCH, J. A. C. HARWOOD, and A. R. PAYNE, Proc. Conf. on Physical Basis of Yield and Fracture, Oxford (1966) (*Inst. Phys. and Phys. Soc.*) p. 144.
13. B. TOMKINS, *Phil Mag.* **18** (1968) 1041.
14. C. E. FELTNER and J. D. MORROW, *Trans ASME* **83** (1961) 15.